

MODIFICATION OF A ONE-DIMENSIONAL LAGRANGIAN GRID FOR MODELING OF DYNAMIC PROCESSES IN PIPELINES OF VARIABLE CROSS SECTION

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Consideration is given to a mathematical model of the nonseparating nonstationary motion of an ideal compressible gas in a pipeline with a variable cross section. A method which enables one to calculate changes in the pressure, density, and velocity at the sites of contraction and expansion of the pipeline without going beyond the scope of a one-dimensional model is presented.

1. Dynamic processes in pipelines are of considerable importance in solving diverse engineering problems. The characteristics of flow that are due to the nonstationarity of the stream must be taken into account, for example, near the sources of pressure jumps, in calculating the strength characteristics of pipe walls, for an accurate calculation of local resistances where parts which disturb the rectilinearity of the generatrix of the pipeline are present, and at the sites of decrease or increase in the pipe diameter. The existing two-dimensional methods for calculating the parameters of a gas in the case of flow in channels are described, for example, in [1, 2]; however, they require that the calculation time be increased, especially when the problem of gas flow is repeatedly solved as part of a more complex problem. One-dimensional schemes, because of the smaller number of computer operations at each time step and the better stability that enables one to calculate with a larger time step, yield a substantial gain in time up to one order of magnitude. Furthermore, in certain cases the two-dimensionality of the flow is used just to calculate more accurately the longitudinal velocity component. Calculations of the parameters of the nonstationary flow of a gas in flowing through the portions of a pipeline with a variable diameter can provide an example of such problems. Below, we describe a method which enables one to calculate changes in these parameters at the sites of contraction and expansion of the pipeline in nonseparating flow without going beyond the scope of a one-dimensional model.

2. Let us consider the region Ω which is part of a pipeline with a variable radius. In order to calculate the flow of a gas in this region, we use the determining system of nonlinear equations for an ideal compressible gas, disregarding mass forces in view of their smallness as compared to pressure forces:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \bar{v}) = 0, \quad (1)$$

$$\frac{\partial \bar{v}}{\partial t} + (\bar{v} \nabla) \bar{v} + \frac{1}{\rho} \nabla P = 0. \quad (2)$$

As the law of conservation of energy we adopt the law of adiabatic flow of the gas

$$P = \left(\frac{\rho}{\rho_0} \right)^\gamma P_0. \quad (3)$$

Considering for the moment the gas flow to be dependent only on the time t and the coordinate x that coincides with the pipeline axis and passing to Lagrangian coordinates [3], we rewrite system (1)–(3) in the form

$$\frac{d\rho}{dt} = -\rho \frac{dv}{dx}, \quad (4)$$

$$\rho \frac{dv}{dt} = -\frac{dP}{dx}, \quad (5)$$

$$\frac{d}{dt} \left(\frac{P}{\rho^\gamma} \right) = 0, \quad (6)$$

where $\partial/\partial t = \partial/\partial t + v\partial/\partial x$ is the total time derivative.

Let us approximate the obtained differential system of equations by a finite-difference scheme as follows: we introduce subdivision of the x axis inside the region Ω by points x_i ($i = 0, \dots, N$); for simplicity these points will be assumed to be uniformly distributed along the x axis at the initial time. The segment between points x_i and x_{i-1} will be called the cell with number i . The points x_i will be considered to be related to a moving gas so that each of them will have a velocity v_i at a given instant of time. Next we will consider that the density ρ_i and the pressure P_i of the gas are determined at the centers of the cells between the points x_i and x_{i-1} . Since the cell moves together with the gas, no substance is transferred through its boundaries. Then, taking into account that the motion is one-dimensional, we can rewrite the equation of conservation of mass (4) for each cell in the form

$$m_i = \rho_i V_i = \rho_i (x_i - x_{i-1}) = \text{const}, \quad i = 1, \dots, N. \quad (7)$$

Here V_i acts as the cell volume. Considering this relation at two different instants of time t^n and t^{n+1} , we obtain

$$\rho_i^{n+1} = \rho_i^n \frac{V_i^n}{V_i^{n+1}} = \rho_i^n \frac{x_i^n - x_{i-1}^n}{x_i^{n+1} - x_{i-1}^{n+1}}, \quad (8)$$

where the superscript denotes the corresponding instant of time while the subscript denotes the number of the point on the x axis.

The coordinates x_i^{n+1} are determined from the formula

$$x_i^{n+1} = x_i^n + \int_{t^n}^{t^{n+1}} v(s) ds. \quad (9)$$

Calculating the integral from (9) according to the left-hand rectangular formula, we arrive at the relation

$$x_i^{n+1} = x_i^n + v_i^n dt, \quad (10)$$

where $dt = t^{n+1} - t^n$.

Since the cells move together with the medium, we must track the conditions of membership of a cell in the calculated region. If the cell is totally beyond the region Ω , we can exclude it from consideration. If, conversely, it penetrates into Ω to a distance larger than its width we must add another cell. By doing so one prevents the cases where all cells will go beyond the region Ω (in the case of constant flow in one direction) or where all cells will shift toward the center of the region (meeting of two compressional waves).

3. We now take into account the radius r of the pipeline as a function of the longitudinal coordinate. Let $r = f(x)$. Instead of one-dimensional cells, we will use three-dimensional ones formed by the pipeline wall and the cross sections through the points x_i . Each such cell is a body of revolution that will be approximated by a frustum of a cone in view of the smallness of the length. Proceeding from the law of conservation of mass, we require that, in motion of such a three-dimensional cell, the mass of the gas be preserved in it. In this case, formula (8) can be rewritten in the form

$$\rho_i^{n+1} = \rho_i^n \frac{(x_i^n - x_{i-1}^n) [r_i^n (R_i^n + r_i^n) + (R_i^n)^2]}{(x_i^{n+1} - x_{i-1}^{n+1}) [r_i^{n+1} (R_i^{n+1} + r_i^{n+1}) + (R_i^{n+1})^2]}, \quad (11)$$

where $R_i = \max(f(x_i), f(x_{i-1}))$ and $r_i = \min(f(x_i), f(x_{i-1}))$ are the radii of the large and small bases of the cone by which the cell is approximated. It should be noted that this formula is also correct in the case where $R_i = r_i$, i.e., the cell is a cylinder. Thus, the density is changed in each cell not only because of the presence of compressional and rarefaction waves in the gas but also because of the change in the radius of the channel along which the gas is flowing. The change in the density in the continuity equation causes the velocity in the equation of motion and the pressure in the equation of state to change.

The remaining equations (5) and (6) are approximated in the following manner: for Eq. (5)

$$v_i^{n+1} = v_i^n - \frac{2dt}{\rho_{i+1}^n + \rho_i^n} \frac{P_{i+1}^n - P_i^n}{\tilde{x}_{i+1}^n - \tilde{x}_i^n}, \quad (12)$$

where $\tilde{x}_i = 0.5(x_i + x_{i-1})$, $i = 1, \dots, N$; for Eq. (6)

$$P_i^{n+1} = P_0 \left(\frac{\rho_i^{n+1}}{\rho_0} \right)^\gamma, \quad i = 1, \dots, N. \quad (13)$$

Equations (11)–(13) are a calculational scheme for modeling the flows of an ideal compressible gas in pipelines with a variable diameter.

It should be noted that this modification of the Lagrangian grid method is suitable for channels in which the change in the shape has no substantial effect on the direction of the average-over-the cross section velocity. The term "substantial" depends on the concrete problem. An example can be provided by the modeling of the gas flow in a branch pipe where the direction of motion changes by 45–90°. In this case, a nonzero radial projection of the velocity that changes the general pattern of the flow appears on the portion of rotation.

4. In order to check the model proposed, we solved the problem of nonstationary escape of a gas from a half-open cavity. A similar problem has been solved in [4] using S. K. Godunov's two-dimensional scheme. We consider three conic closed cavities of length L (cylindrical (Fig. 1a), divergent (Fig. 1b), and convergent (Fig. 1c) cavities) filled, at the initial instant of time, with compressed gas. Upon the instantaneous breaking of the shell on the section of the cavity, we investigated fluctuations of the gas pressure at its bottom. The length of the region was selected from the condition of equality of the time in which the distur-

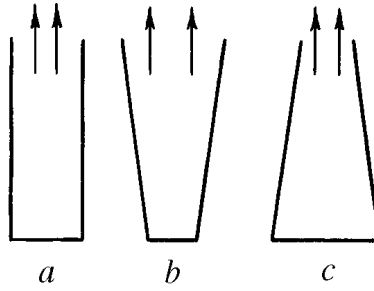


Fig. 1. Calculated regions of the flow.

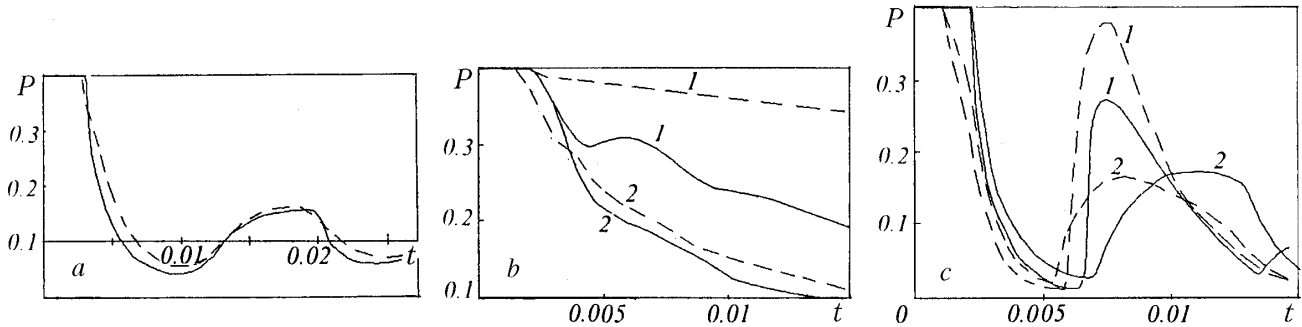


Fig. 2. Standard calculation of the flow in cylindrical (a), convergent (b) [1] $d_{av} = 0.11$; 2) 0.33], and divergent (c) [1] $d_{av} = 3$; 2) 1.5] regions. P , MPa; t , sec.

bance wave reaches the bottom of the cavity obtained in this work and the time obtained in [4]. To check the stability and reliability of the model, we performed the calculations in several stages with a gradual decrease in the initial step of the grid. Once the decrease in the step ceased to have an effect on the solution, we stopped the calculations. But for this minimum step, too, the total time of calculation according to the proposed scheme turned out to be shorter than the time obtained according to the Godunov method.

We adopted the following initial data:

cavity length $L = 1$ m,

ratio of the cavity length to the average diameter $L/r = 3$ for a cylindrical cavity and $L/r = 10$ for a conic cavity,

pressure in the cavity $P = 0.4$ MPa,

initial velocity of sound $c = 300$ m/sec,

adiabatic exponent $\gamma = 1.25$,

pressure of the ambient medium $P_0 = 0.1$ MPa.

The calculations according to the method proposed showed good qualitative agreement between the obtained values of the gas parameters and the results from [4]. An exception is the initial period of escape until the rarefaction wave reaches the bottom. However, in this time interval, the velocity of escape is the highest. This maximum of the velocity corresponds to the largest values of its radial component, especially near longitudinal walls where the multidimensionality of the flow manifests itself earlier and to the greatest extent, which reduces the accuracy of the calculations. Plots of pressure fluctuations at the bottom of the cavity are presented in Fig. 2. The results of calculations by a modified Lagrangian-grid method are denoted by the solid lines, while those according to the Godunov two-dimensional scheme are shown by the dashed lines. On all the plots the time is calculated in seconds. Figure 2a presents pressure fluctuations at the bottom for a cylindrical region as the standard of the error of the method proposed; Fig. 2b and c shows pressure fluctuations for divergent and convergent regions respectively.

Curves 1 and 2 in Fig. 2b reflect the cases where the ratio of the diameters on the section and at the bottom of the cavity d_{av} is equal to 0.11 and 0.33 respectively. It is obvious that the presence of convergence leads to a decrease in the amplitude of the pressure fluctuations just as in [4]. We were unable to achieve the nonoscillating process of escape of the gas (dashed curve 1) in the calculations; however, it is necessary to note that in this case the boundary diameters of the cavity differ by approximately a factor of 10, and this yields a substantially multidimensional character of the flow which cannot be described within the framework of a one-dimensional model. However, already for a diameter ratio of 0.33 the results are in good agreement.

Curves 1 and 2 in Fig. 2c reflect the cases where the ratio of the diameters on the section and at the bottom of the cavity is equal to 3 and 1.5 respectively. As is indicated in [4], an increase in the divergence of the cavity leads to an increase in the amplitude of pressure fluctuations, up to the shock loading of the bottom. The similar escape of the gas (Fig. 2c, curve 1) is also substantially multidimensional, and the modified Lagrangian-grid method yields just qualitatively acceptable results. However, already in the case of the average divergence where the condition of the nonseparating motion of the gas along the channel walls is ensured [5], calculation results for the two-dimensional scheme and the proposed method coincide.

It should be noted that the problem considered is not only multidimensional but substantially nonstationary as well, especially in the case of a divergent region, since a pressure drop of 0.4 MPa at a rather small length is not characteristic of the flow in pipelines. In this connection, the above disagreement of the results is maximum for this class of problems; hence for the flows in pipelines where subsonic nonseparating flows are realized, the accuracy of the calculations will be higher.

The proposed modification of the Lagrangian-grid method yields good accuracy of calculations comparable with two-dimensional methods for the cases where the relative change in the diameter of the pipeline is within 0.3–1.5 of the average diameter and the flow is nonseparating in nature and also requires less computer time in calculations.

NOTATION

ρ , density; v , velocity; P , pressure of the gas; ρ_0 and P_0 , parameters of the gas in the atmosphere; γ , adiabatic exponent; t , time; x , coordinate along the region of gas flow; Ω , investigated region of the flow; L , length of the pipeline; x_i , coordinates; v_i , velocities of the moving boundaries of the cells of the calculational scheme; m_i , ρ_i , and P_i mass, density, and pressure of the gas inside the calculational cells; t^n , running instant of time; V_i , volume of the cell with the number i ; $dt = t^{n+1} - t^n$, time step; N , number of points in the subdivision of the x axis; s , variable of integration.

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